

# Charge and stability of strange quark matter at finite temperature

Yun Zhang<sup>1,3</sup> and Ru-Keng Su<sup>2,3</sup>

<sup>1</sup>*Surface Physics Laboratory (National Key Laboratory), Fudan University,  
Shanghai 200433 , P. R. China*

<sup>2</sup>*China Center of advanced Science and Technology (World Laboratory)  
P. O. Box 8730, Beijing 100080, P. R. China*

<sup>3</sup>*Department of physics, Fudan University, Shanghai 200433 , P. R. China*

## Abstract

By means of the quark mass density- and temperature- dependent model, it is found that the negative charge and the higher strangeness fraction are in favor of the stability of strange quark matter at finite temperature. A critical baryon number density  $n_{Bc}$  has been found. The strange quark matter is unstable when  $n_B < n_{Bc}$ . The charge and strangeness fraction dependence of stability for strange quark matter is addressed.

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# I. INTRODUCTION

It has been found from the theoretical study that the introduction of strange quarks into a plasma with two flavors could lower the Fermi energy of the system and thus the mass of quark matter [1]. If its mass is lower than the mass of hyperonic matter with the same strangeness fraction, or even lower than the non-strange nucleonic matter, the strange quark matter (SQM) would be the true ground state of QCD. Therefore, to search the SQM in the laboratory is an important task for recent heavy ion collision experiments at ultrarelativistic energy [2].

The essential problem for detectability of SQM in heavy ion experiments is to study its stability during the formation of quark gluon plasma (QGP). Many properties of stability of SQM, for example, the "stability window" [3–5], the charge-dependence [6–9] and the strangeness fraction-dependence [10–14] of stability have been widely discussed by many authors. They employed different models to investigate this problem. But unfortunately, many conclusions are different, in particular, part of them, say, the charge dependence is contrary.

The electric charge is of vital importance to the experimental searches of SQM. By means of MIT bag model, Jaffe and his co-workers [3,6,7] suggested that the SQM is slightly positively electric charged. They considered strong and weak decay by nucleon and hyperon emission together and concluded that the stable SQM will have a low but positive charge to mass ratio. Using the same model as that of Jaffe et al. but considering the initial condition of possible strangelet production in relativistic heavy ion collision carefully, Greiner and his co-workers [7,12] argued that strangelets are most likely highly negatively charged. They also emphasized that the stability of infinite SQM depends on the bag constant  $B$ . An infinite SQM, treated as a gas of noninteracting quarks, is absolutely stable for  $B^{1/4} \simeq 145\text{MeV}$ , and metastable for  $B^{1/4} \simeq 150 - 200\text{MeV}$ , but unstable for larger bag constants. Of course, this conflict of conclusions is limited in the framework of MIT bag model and at zero temperature.

Employing another model, namely, the quark mass density-dependent (QMDD) model, Peng and his co-workers reconsidered this problem [9]. They introduced a critical density and proved that SQM cannot maintain its flavor equilibrium, and thus no longer be stable under this critical density. They found that proper negative charges can lower the critical density and thus make the SQM become more stable. Their conclusion is not surprised if one notice the confinement mechanism of QMDD model. In QMDD model, the mass of  $u, d$  and  $s$  quarks are given by [15–17,4,9,7,3]

$$m_q = \frac{B}{3n_B}, \quad (q = u, d, \bar{u}, \bar{d}), \quad (1)$$

$$m_{s,\bar{s}} = m_{s0} + \frac{B}{3n_B}, \quad (2)$$

where  $n_B$  is the baryon number density,  $m_{s0}$  is the current mass of the strange quark matter and  $B$  is the vacuum energy density inside bag (bag constant). The mechanism of confinement can be mimicked through the hypothesis Eqs.(1) and (2). The masses of quarks become infinitely large as the volume increases to infinity or the density decreases to zero. So that the vacuum is unable to support it [4,17]. Noting that the boundary condition of confinement of MIT bag model corresponds to the zero quark mass inside the bag but

infinity at the boundary or outside the bag, we see that the confinement mechanisms are similar to these two models. Due to this correspondence, as pointed out by Benvenuto and Lugones [4], in almost all cases, the properties of SQM given by QMDD model are nearly the same as those obtained in the MIT bag model. The conclusion of Peng et al. supports the result given by Greiner et al.

This paper evolves from an attempt to extend these discussions to finite temperature. The reasons of this extension are: at first, the SQM might be possible and formed during the phase transition from hadronic matter to a deconfined QGP. From lattice QCD calculations, the phase transition of deconfinement will happen at critical temperature. Secondly, as was pointed out by our previous papers [5,18], QMDD model cannot be used to explain the process of the quark deconfinement phase transition because the quark confinement is permanent in this model. Instead of this model, a quark mass density- and temperature-dependent (QMDTD) model in which the quark confinement is not permanent has been suggested by us [5,18]. As its application, we hope to employ QMDTD model to discuss the charge and strangeness fraction dependences of SQM at finite temperature.

The organization of this paper is as follows: In next section, we review the QMDTD model and give the main formulae which are necessary for studying the thermodynamical behaviors of SQM. In section 3, we give the result of numerical calculations for QMDTD model and investigate the stability of hot SQM in detail. Our conclusions are summarized in the last section.

## II. QUARK MASS DENSITY- AND TEMPERATURE-DEPENDENT MODELS

To overcome the difficulties of QMDD model which appear at finite temperature, in refs. [5,18], based on the Friedberg-Lee model [19], we suggested a QMDTD model. The basic difference between our model and QMDD model is that, instead of a constant  $B$  in Eqs.(1) and (2), we argued that  $B$  must be a function of temperature and choose  $B(T)$  as

$$B = B_0 \left[ 1 - \left( \frac{T}{T_c} \right)^2 \right], 0 \leq T \leq T_c \quad (3)$$

$$B = 0, T > T_c, \quad (4)$$

where  $B_0$  is bag constant at zero temperature and  $T_c$  is the critical temperature of quarks deconfinement. Substituting Eq.(3) into Eqs.(1) and (2), we find the masses of  $u, d, s$  quarks and their corresponding anti-quarks as

$$m_q = \frac{B_0}{3n_B} \left[ 1 - \left( \frac{T}{T_c} \right)^2 \right], \quad (q = u, d, \bar{u}, \bar{d}), \quad (5)$$

$$m_{s,\bar{s}} = m_{s0} + \frac{B_0}{3n_B} \left[ 1 - \left( \frac{T}{T_c} \right)^2 \right], \quad (6)$$

when  $0 \leq T \leq T_c$ . When  $T \geq T_c$ , the masses of quarks are independent of temperature:  $m_{u,d,\bar{u},\bar{d}} = 0$  and  $m_{s,\bar{s}} = m_{s0}$ . Due to the difference between Eqs.(1),(2) and Eqs.(5),(6), as was proved in ref. [5], we found that when  $n_B$  approaches to zero, the temperature  $T \rightarrow T_c$  in our model, but diverges in QMDD model. In QMDTD model, the masses of  $u, d$  quarks

become zero (or constant for strange quarks) when the temperature approaches  $T_c$ . This means that the quarks can be deconfined. Here, the vacuum energy density  $B$  is a decreasing function as temperature increases. As was pointed out by Greiner et al., the chosen values of  $B$  affect the stability of SQM remarkably [8]. Therefore, it is of interest to consider the effect of  $B(T)$ .

Treating the infinite SQM as a gas of noninteracting quarks and electrons [6,9], we find that the thermodynamical potential density of SQM is

$$\Omega = \sum_i \Omega_i = - \sum_i \frac{g_i T}{(2\pi)^3} \int_0^\infty dk \frac{dN_i}{dk} \ln \left( 1 + e^{-\beta(\varepsilon_i(k) - \mu_i)} \right), \quad (7)$$

where  $i$  stands for  $u, d, s$  (or  $\bar{u}, \bar{d}, \bar{s}$ ) and the electron  $e(e^+)$ ,  $g_i = 6$  for quarks and antiquarks,  $g_i = 2$  for  $e$  and  $e^+$ .  $\varepsilon_i(k) = \sqrt{m_i^2 + k^2}$  is the single particle energy and  $m_i$  is given by Eqs.(5),(6) for quarks and antiquarks.  $\mu_i$  is the chemical potential (for antiparticle  $\mu_{\bar{i}} = -\mu_i$ ). The number density of each particle can be obtained from eq.(7) by means of

$$n_i = - \frac{1}{V} \left. \frac{\partial \Omega}{\partial \mu_i} \right|_{T, n_B}, \quad (8)$$

and we find

$$\Delta n_i = n_i - n_{\bar{i}} = \frac{g_i}{(2\pi)^3} \int_0^\infty d^3k \left( \frac{1}{\exp[\beta(\varepsilon_i - \mu_i)] + 1} - \frac{1}{\exp[\beta(\varepsilon_i + \mu_i)] + 1} \right). \quad (9)$$

For SQM, the baryon number density satisfies

$$n_B = \frac{1}{3}(\Delta n_u + \Delta n_d + \Delta n_s). \quad (10)$$

The electric charge density  $Q$  of SQM reads

$$Q = \frac{2}{3}\Delta n_u - \frac{1}{3}\Delta n_d - \frac{1}{3}\Delta n_s - \Delta n_e, \quad (11)$$

and the light quarks are converted to strange quarks through the weak processes [9,16]

$$u + d \longleftrightarrow u + s, s \rightarrow u + e^- + \bar{\nu}_e, d \rightarrow u + e^- + \bar{\nu}_e, u + e^- \rightarrow d + \nu_e. \quad (12)$$

Neglecting the contribution of neutrinos [3,4,16,17], we obtain the conditions of chemical equilibrium from the reactions (12) as

$$\mu_s = \mu_d, \quad (13)$$

$$\mu_s = \mu_u + \mu_e. \quad (14)$$

Noting that the chemical potentials of particles and anti-particles are determined by Fermi distribution Eq.(9), we see that Eqs.(5), (6), (9), (10), (11), (13) and (14) form an equation group and can be solved self-consistently. The solution of this equation group determines the chemical stability of SQM. The results of numerical calculation will show in the next section. Here we want to point out that at a fixed temperature  $T$  and for a fixed value of charge density  $Q$  this equation group which determines the configuration of SQM has no solution under a critical baryon number density  $n_{Bc}$ . This means that the SQM is unstable when  $n_B < n_{Bc}$ . With the values of  $Q$  changed and the temperatures limited between  $0 \leq T \leq T_c$ , the condition  $n_B \geq n_{Bc}$  determines a stable area which might be useful for the detectability of SQM in experiments of high energy heavy ion collisions.

### III. CONCLUSION AND DISCUSSION

The numerical calculations of the equation group mentioned above have been done with the parameters set:

$$B_0 = 170\text{MeVfm}^{-3}, m_{s0} = 150\text{MeV}, T_c = 170\text{MeV}. \quad (15)$$

Firstly, we study a SQM system with neutrality charge  $Q = 0$ , for a given  $n_B$ , the fractions of different quarks are given by

$$F_{q(q=u,d,s)} = \frac{\Delta n_q}{3n_B}. \quad (16)$$

At a fixed temperature  $T = 100\text{MeV}$ , the  $u, d, s$  quarks fraction are drawn in Fig.1 by solid line, dashed line and dotted line, respectively. At high baryon number density, all of the  $u, d, s$  quarks tend to become a triplicate. When the density becomes lower,  $u$  fraction increases and  $s$  fraction decreases monotonously. We see from this figure that the fractions of  $u, d, s$  quarks satisfy an inequality:

$$F_d > F_u > F_s. \quad (17)$$

In particular, we would like to emphasize that, as shown in Fig.1, three different quark fraction curves stop at a critical baryon number density ( $n_{Bc} = 8.28 \times 10^{-3}\text{fm}^{-3}$ ). One can not find real roots for  $F_d, F_u$  and  $F_s$  provided  $n_B < n_{Bc}$ . At  $n_{Bc}$ ,  $F_s = 0.107 \neq 0$ . In ref. [9], the authors defined the critical density by the condition  $F_s = 0$ . Of course, the SQM can not maintain the chemical equilibrium if no strange quarks exist. But as shown in Fig.1, this is only a sufficient but is not the necessary condition. Because, one cannot find solution in the ranges  $0 < F_s < 0.107$ .

To compare the QMDD model in which  $B$  is a constant and our QMDTD model in which  $B$  satisfies Eqs.(3) and (4), we plot the critical density  $n_{Bc}$  vs temperature  $T$  with neutral charge density  $Q = 0$  in figure 2, where the dashed line and the solid line refer to QMDD model and QMDTD model, respectively. We see that the curve given by QMDD model can never be zero even the temperature increases to  $T > T_c$ . It can easily be understood if we notice that  $m_q$  approaches to infinity when  $n_B \rightarrow 0$  from Eqs.(1) and (2). To excite a particle with infinite mass must spend infinite energy and then infinite temperature. Because of  $n_{Bc} \neq 0$ , when  $n_B < n_{Bc}$ , the system would still be unstable even at very high temperature for QMDD model. But for QMDTD model, we see from the solid line that  $n_{Bc} = 0$  at  $T = T_c = 170\text{MeV}$ . It means that the SQM is completely stable and may be detected from high energy heavy ion collision.

Now we are in a position to address the charge dependence of stability for SQM. The critical density vs temperature curves with various charges are shown in figure 3, where the solid line, dotted line and dashed line refer to  $Q = 0, -0.1\text{efm}^{-3}, -0.2\text{efm}^{-3}$ , respectively. We see that the critical density  $n_{Bc}$  decreases rapidly and the stable area expands widely when  $Q$  decreases to more negative. So the negative charge is in favor of the stability of SQM. Our result supports the conclusion given by Greiner et al. [8] and extends their result to finite temperature.

To study the effect of the strangeness fraction, we show the same curves as that of Fig.1 but for  $Q = -0.2\text{efm}^{-3}$  in figure 4. Comparing Fig.1 with Fig.4, we find that all of three

curves tend to become a triplicate for  $Q = -0.2\text{efm}^{-3}$  at high density, too. And there are two major differences between these two figures, firstly, the critical density decreases from  $8.28 \times 10^{-3}\text{fm}^{-3}$  to  $5.02 \times 10^{-4}\text{fm}^{-3}$  when charge density  $Q$  changes from neutrality to negativity  $-0.2\text{efm}^{-3}$ ; secondly, Eq.(17) becomes

$$F_d > F_s > F_u. \quad (18)$$

The fraction of  $s$  quarks is higher than the fraction of  $u$  quarks. This is of course reasonable because  $s$  quark has negative charge and  $u$  quark positive. The critical density  $n_{Bc}$  decreases as the  $F_s$  increases. The higher strangeness fraction is in favor of the stability of SQM. To illustrate this result more transparently, using the expression of strangeness fraction

$$f_s = \frac{\Delta n_s}{n_B}, \quad (19)$$

which is usually employed by many references, we draw the critical strangeness fraction  $f_{sc}$  vs temperature  $T$  curves in figure 5, where the solid line and dashed line refer to  $Q = 0$  and  $-0.2\text{efm}^{-3}$ , respectively. The  $f_{sc}$  is strangeness fraction corresponding to the critical density  $n_{Bc}$  at the same conditions of charge and temperature. We see from Fig. 5 the critical strangeness fraction increases with increasing temperature. The stable area corresponds to the larger negative charge and higher strangeness fraction.

Finally, we hope to investigate the stability of SQM from another view point. Instead of considering the chemical equilibrium, we study the behavior of chemical potentials for electron and  $u$  quark, respectively. The reason for this choice is that the electron ( $u$  quark) is the particle with minimum mass but negative (positive) charge in our model. The smaller the particle mass is, the lower the chemical potential will be. When chemical potential tends to be negative, the system becomes unstable because the Fermi surface is negative in this case. Therefore, we can use the conditions  $\mu_e \geq 0$ ,  $\mu_u \geq 0$  to determine the stability of SQM.

The chemical potential of electron  $\mu_e$  vs baryon number density  $n_B$  at a fixed temperature  $T = 100\text{MeV}$  with various positive charge density  $Q = 1 \times 10^{-4}, 1 \times 10^{-3}, 3 \times 10^{-3}, 5 \times 10^{-3}$  and  $7 \times 10^{-3}\text{efm}^{-3}$  are drawn in figure 6. We see that  $\mu_e$  becomes negative and the system becomes unstable when  $Q < 0.001\text{efm}^{-3}$ . Similarly, the chemical potential of  $u$  quark  $\mu_u$  vs baryon number density  $n_B$  at a fixed temperature  $T = 100\text{MeV}$  with various negative charge density  $Q = -0.2, -0.322, -0.4, -0.6$  and  $-0.8\text{efm}^{-3}$  are drawn in figure 7. We see from this figure that when  $Q < -0.322$ , the curve will pass through a negative area and thus SQM is unstable. Therefore, at temperature  $T = 100\text{MeV}$ , the charge density must be limited to  $0.001\text{efm}^{-3} > Q > -0.322\text{efm}^{-3}$  for stable SQM.

#### IV. SUMMARY

In summary, by means of the QMDTD model, we find that the negative charge and the higher strangeness fraction are in favor of the stability of SQM at finite temperature. This result is in agreement with that given by Greiner et al. [8] at zero temperature, but we extend their conclusion to finite temperature. A critical baryon number density  $n_{Bc}$  under which the SQM is unstable has been obtained. We find that  $n_{Bc}$  decreases rapidly when charge density

$Q$  becomes more negative and approaches to zero at the deconfinement temperature  $T_c$ . By using the chemical potentials of electron and  $u$  quark, we suggest a method to determine the limit of charge density for the stability of SQM. We hope the stable areas given by our model can have impact on the detectability of SQM in future heavy ion experiments.

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## VI. FIGURE CAPTIONS

Fig.1 The quark fraction  $F_q$  as a function of baryon number density  $n_B$  with neutral charge,  $F_u, F_d$  and  $F_s$  are represented by solid, dashed and dotted lines, respectively. Here  $F_d > F_u > F_s$ .

Fig.2 The critical density  $n_B$  as a function of temperature  $T$  with neutral charge, the dashed line is for QMDD model and the solid line is for QMDTD model.

Fig.3 The critical density  $n_B$  as a function of temperature  $T$  with various charge density  $Q = 0, -0.1$  and  $-0.2\text{efm}^{-3}$ , they are represented by solid, dotted and dashed lines, respectively.

Fig.4 The quark fraction  $F_q$  as a function of baryon number density  $n_B$  with a negative charge density  $Q = -0.2\text{efm}^{-3}$ ,  $F_u, F_d$  and  $F_s$  are represented by solid, dashed and dotted lines, respectively. Now  $F_d > F_s > F_u$ .

Fig.5 The critical strangeness fraction  $f_s$  as a function of temperature  $T$ . The solid line is for  $Q = 0$  and the dashed line is for  $Q = -0.2\text{efm}^{-3}$ .



Fig.6 The chemical potential of electron  $\mu_e$  as a function of baryon number density  $n_B$  with the positive charge density  $Q = 1 \times 10^{-4}, 1 \times 10^{-3}, 3 \times 10^{-3}, 5 \times 10^{-3}$  and  $7 \times 10^{-3} \text{efm}^{-3}$ . At a fixed temperature  $T = 100 \text{MeV}$ , they are represented by solid lines respectively.

Fig.7 The chemical potential of  $u$  quark  $\mu_u$  as a function of baryon number density  $n_B$  with the negative charge density  $Q = -0.2, -0.322, -0.4, -0.6$ , and  $-0.8 \text{efm}^{-3}$ . At a fixed temperature  $T = 100 \text{MeV}$ , they are represented by solid lines respectively.